Office Hours: Mon 12/4 2-3PM

Thu 12/7 ~~2-3PM~~ 11-12:30PM

Mon 12/11 1-2:30PM

Thu 12/14 12:30 - 2PM

Final Exam: Fri 12/15 3:30 PM

Recap: arcsin(x), arccos(x), arctan(x)

Falling Ladder (arccos(x)), Viewing Angle (arctan(x))

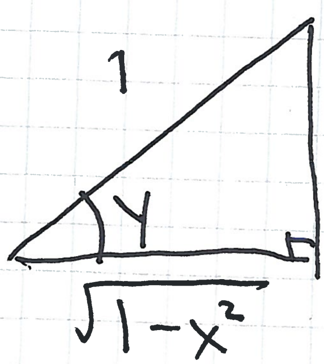
Derivatives of these:

Ex. $\arcsin(x) = y$

$x = \sin(y)$

Inverse function trick

$\frac{dx}{dy} = \cos(y)$

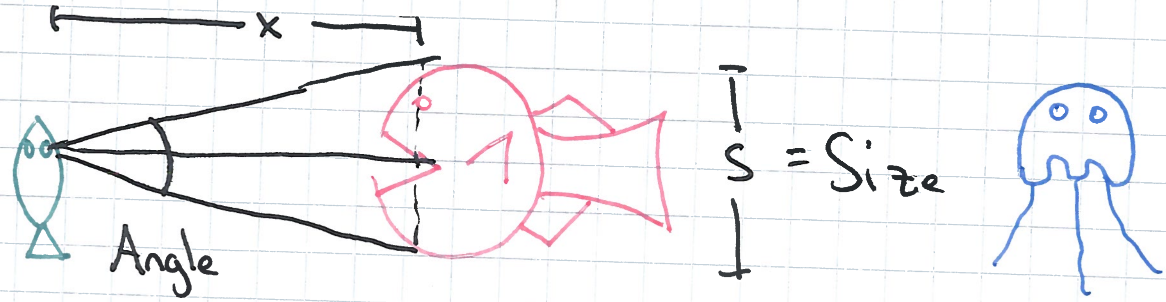


$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\cos(\arcsin(x))}$

$\cos(y) = \sqrt{1-x^2}$

$$\boxed{d(\arctan(x)) = \frac{1}{1+x^2}}$$

$= \frac{1}{\sqrt{1-x^2}}$



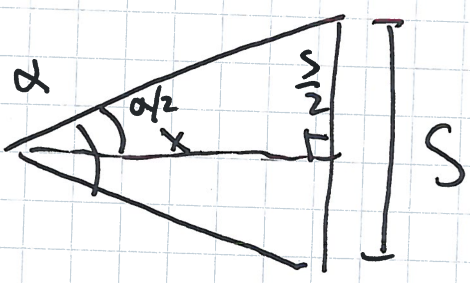
Constant velocity $v = -\frac{dx}{dt}$

Hypothesis: Escape response triggered when $\frac{d\alpha}{dt} > K_{crit}$

Constant "critical threshold"

K_{crit} small = skittish.

Q1: Express $\frac{d\alpha}{dt}$ in terms of S, x, v



$$\tan\frac{\alpha}{2} = \frac{S/2}{x} = \frac{S}{2x}$$

$$\frac{\alpha}{2} = \arctan\left(\frac{S}{2x}\right)$$

$$\alpha = 2\arctan\left(\frac{S}{2x}\right)$$

$$\frac{d\alpha}{dt} = 2\,\frac{1}{1+\left(\frac{S}{2x}\right)^2}\cdot\frac{d\left(S/2x\right)}{dt}$$

$$= \cancel{2}\cdot\frac{1}{1+\left(\frac{S}{2x}\right)^2}\cdot\left(-\frac{S}{\cancel{2}x^2}\right)\cdot(-v)$$

$$= \boxed{\frac{vS}{x^2+\frac{S^2}{4}}}$$

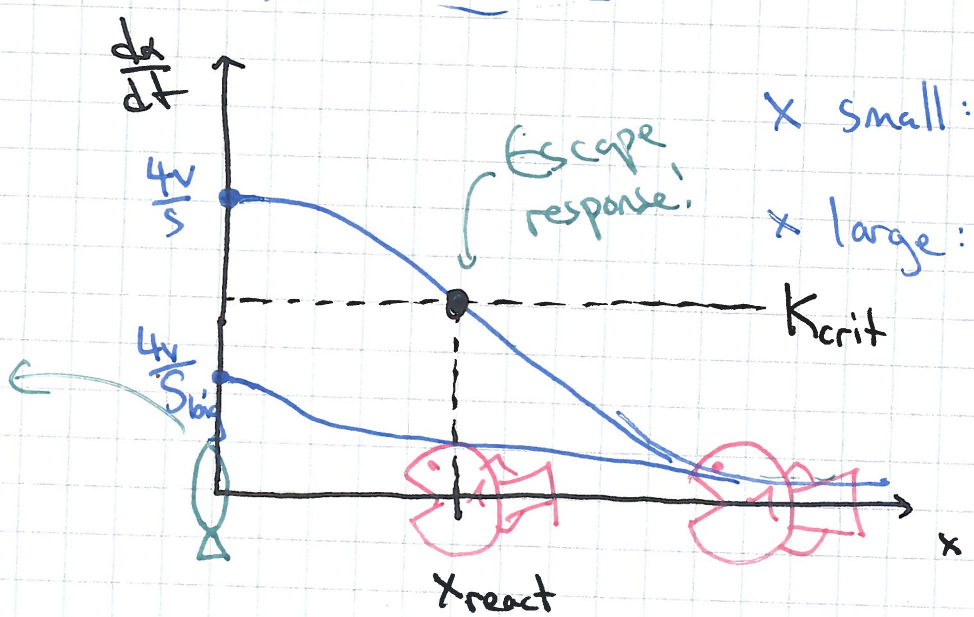
$$\frac{da}{dt} = \frac{vS}{x^2 + \frac{S^2}{4}}$$

Q: When is this larger than the constant K_{crit}?

v large $\rightsquigarrow$ $\frac{da}{dt}$ large

S large $\rightsquigarrow$ not clear.

Q2: Plot $\frac{da}{dt}$ against x. (Assume S fixed)



x small: $\frac{vS}{\left(\frac{S^2}{4}\right)} = \frac{4v}{S}$

x large: $\frac{vS}{x^2}$

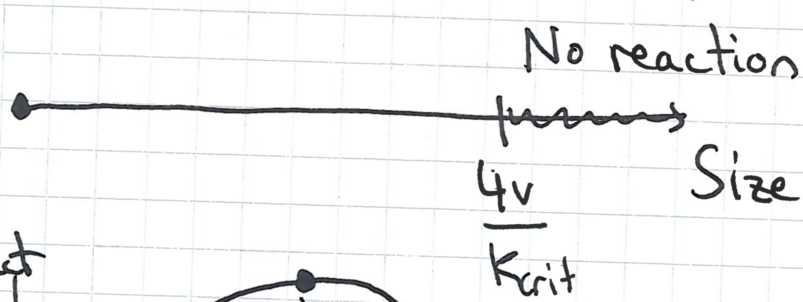
If $\frac{4v}{S} < K_{crit}$ then no reaction!

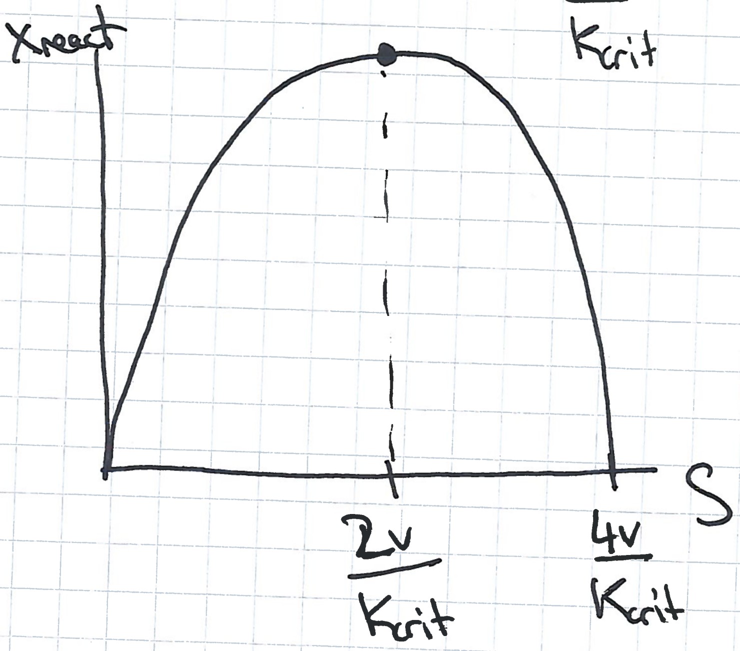
Q: What is x_{react}? $\frac{vS}{x^2 + \frac{S^2}{4}} = K_{crit}$

If $\frac{4v}{S} < K_{crit}$ then this is negative

$$x_{react} = \sqrt{\frac{Sv}{K_{react}} - \frac{S^2}{4}} = \sqrt{S\underbrace{\left(\frac{v}{K_{react}} - \frac{S}{4}\right)}}$$

What S maximizes x_{react}?





$$\frac{d\alpha}{dt} = \frac{vS}{x^2 + \frac{S^2}{4}}$$

Graph $\frac{d\alpha}{dt}$ against S

S small: $\frac{d\alpha}{dt} \approx \frac{vS}{x^2}$

S large: $\frac{d\alpha}{dt} \approx \frac{vS}{\frac{S^2}{4}}$

$= \frac{4v}{S}$